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LRS Bianchi type-I Universe in *F(T)* Theory of Gravity-II

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Abstract:

In this paper, we have investigated the spatially homogeneous and anisotropic Locally Rotationally Symmetric (LRS) Bianchi type-I universe in F(T) theory of gravity. We have examined some models corresponding to the study of equation of state parameter representing the different phases of the universe. For this, we have considered matter, radiation and dark energy. An attempt has been made to retain Sharif and Rani's (2011) forms of the various quantities. Our results are analogous to the result obtained by Sharif and Rani (2011).

Keywords: *F(T) gravity, LRS Bianchi type-I universe, Equation of State parameter.*

1 Introduction :

The most recent observational data states that our universe is in an accelerated expansion phase and further such expansion is driven by mysterious energy having negative pressure known as Dark Energy (DE). The Supernova Iaexperiment [Perlmutter*et al.* (1997),(1998), (1999), Riess*et al.* (1998)] provides basic stand for the accelerating expansion of the universe. Other observations of anisotropies in the cosmic microwave background (CMB) radiation are studied with the data obtained from WMAP satellite[Bennett *et.al.* (2003)] and large scale structure [Verde *etal.* (2002)]. It is observed that most of the part of universe isoccupied by dark matter and dark energy. The cosmological constant is considered as one of the candidate that is responsible for dark energy. Einstein introduced the concept of dark energy by the introduction of the cosmological constant.

An alternate approachfor the study of the universe has been provided by the modified theories of gravity.Some authors use theidea of the existence of a dark energy while others believe in modification of General Relativity. Interesting alternative to General Relativity is*F(T)* gravitywhich gathers considerable attention to give good explanation of late time acceleration [Ferraro and Fiorini (2007), Bengochea and Ferraro (2009), Linder (2010)].Without the introduction of any DE component,*F(T)* gravity can explain the accelerated expansion of the universe [Myrzakulov (2011), Dent *et al.* (2011)]. This model utilizes the Weitzenböck connection, which has only torsion which is responsible for accelerating expansion of the universe [Myrzakulov (2011)]. The *F(T)* theory of gravityproduces equations of order two in field derivatives [Abbas *et al.* (2015), Nassur*et al.* (2015), Geng*et al.* (2015), Das *et al.* (2015)].

In the presence of anisotropic DE, many authors have studied the Bianchi Type-I (B-I) model. A Bianchi Type-I ΛCDM cosmological model where DE component preserves non-dynamical character, but yield anistropicvaccume pressure has been constructed by Rodringues (2008). Many Researchers worked on this model using various parameters and theories [Bali and Kumawat (2008), Amirhashchi (2011), Yadav and Saha (2012), Adhav(2011a), Adhav(2011b), Adhav (2012), Adhav*at el.* (2011)].

In this paper, we have investigated*F(T)* models by using LRS B-I space time. The paper has been arranged as follows: In section 2,we have presented some basics of tele-parallel gravity. In section 3, formulation of field equations for LRS B-I has been done. By using EoS parameter, the construction of some *F(T)* models with different cases of perfect fluid has been examined in section 4. Finally, in section 5, we have summarized and with the conclusion.

2. *F(T)* gravity :

The *F(T)* theory of gravity is defined in Weitzenböck space time with line element as specified by

$$ds^2 = g_{\mu\upsilon} dx^\mu dx^\upsilon \quad ,(1)$$

where $g_{\mu\upsilon}$ are the symmetric components having 10 degrees of freedom.

The theory can be described in space-time or in tangent space through the matrix called tetrad as follows

$$ds^2 = g_{\mu\upsilon} dx^\mu dx^\upsilon = \eta_{ij} \theta^i \theta^j , \qquad (2)$$

$$dx^\mu = e_i{}^\mu \theta^i \quad , \qquad \theta^i = e^i{}_\mu dx^\mu , \ (3)$$

where $e_i e_j = n_{ij}$, where $n_{ij} = diag(1,-1,-1,-1)$ is the Minkowski metric

The square root of the metric determinant is given by $\sqrt{-g} = \det\left[e^i{}_\mu\right] = e$ and the metric $e^a{}_\mu$ are called tetrad representing the dynamic fields of theory.

The Weitzenböck connection is defined by using these fields as

$$\Gamma^\alpha_{\mu\upsilon} = e_i{}^\alpha \partial_\upsilon e^i{}_\mu = -e^i{}_\mu \partial_\upsilon e_i{}^\alpha \qquad .(4)$$

By using this connection the main geometrical objects of space-time are constructed.

Thecomponents of the tensor torsion are defined by the antisymmetric part of this connection as

$$T^\alpha{}_{\mu\upsilon} = \Gamma^\alpha_{\upsilon\mu} - \Gamma^\alpha_{\mu\upsilon} = e_i^\alpha \left(\partial_\mu e^i{}_\upsilon - \partial_\upsilon e^i{}_\mu\right) \quad . (5)$$

We can define two tensor in this gravity, the contorsion tensor and antisymmetric tensor as

$$K^{\mu\upsilon}{}_\alpha = -\frac{1}{2}\left(T^{\mu\upsilon}{}_\alpha - T^{\upsilon\mu}{}_\alpha - T_\alpha{}^{\mu\upsilon}\right), \qquad (6)$$

$$S_\alpha{}^{\mu\upsilon} = \frac{1}{2}\left(K^{\mu\upsilon}{}_\alpha + \delta^\mu_\alpha T^{\beta\upsilon}{}_\beta - \delta^\upsilon_\alpha T^{\beta\mu}{}_\beta\right) \ (7)$$

From equations (5) - (7) the torsion scalar is defined as

$$T = T^\alpha{}_{\mu\upsilon} S_\alpha{}^{\mu\upsilon} \ . \qquad (8)$$

The action of the *F(T)* gravity is defined by generalising the teleparallel theory as

$$S = \int e\left[F(T) + L_{Matter}\right] d^4x \ , \qquad (9)$$

where*F(T)* is an algebraic function of torsion scalar *T*.

The field equations of *F(T)* theory of gravity for the action (9)reads

$$\left[e^{-1}\partial_\mu\left(eS_i{}^{\mu\upsilon}\right) - e_i^\lambda T^\alpha{}_{\mu\lambda} S_\alpha{}^{\upsilon\mu}\right] F_T + S_i{}^{\mu\upsilon}\partial_\mu(T) F_{TT} + \frac{1}{4} e_i^\upsilon F = \frac{1}{2} k^2 e_i^\alpha T_\alpha^\upsilon \quad , (10)$$

where $k^2 = 8\pi G$, $F_T = \dfrac{dF}{dT}$.

In this case, the energy momentum tensor which is contribution of the interaction with the matter fields is defined as

$$T_\mu^\upsilon = diag\left(\rho_m \ , -p_m \ , -p_m \ , -p_m\right), \quad (11)$$

where ρ_m and p_m are energy density and pressure of matter.

3. The Field Equations:

The line element for spatially homogenous and anisotropic LRS Bianchi type-I space-time is described by [Sharif and Zubair (2010)]

$$ds^2 = dt^2 - A^2(t) dx^2 - B^2(t)\left(dy^2 + dz^2\right), \quad (12)$$

where A and B are the function of cosmic time t only.

For this metric

$$e^i_\mu = diag(1, A, B, B), \quad e_i^\mu = diag\left(1, A^{-1}, B^{-1}, B^{-1}\right) (13)$$

The Hubble parameter H_i in the direction of x, y and z axes respectively are given as

$$H_1 = \frac{\dot{A}}{A} \ , \quad H_2 = H_3 = \frac{\dot{B}}{B} \ . \qquad (14)$$

By using equations (5) - (7), the torsion tensor in equation (8)has the form

$$T = -2\left(2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2}\right). \qquad (15)$$

By using equations (12) – (15), for $i = 0 = \upsilon$ and $i = 1 = \upsilon$, the above field equations (10) of*F(T)* theory of gravity are obtained as

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$$F-4\left(2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right)F_T=2k^2\rho_m, \qquad (16)$$

$$4\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{\ddot{B}}{B}\right)F_T-16\frac{\dot{B}}{B}\left[\frac{\dot{B}}{B}\left(\frac{\ddot{A}}{A}-\frac{\dot{A}^2}{A^2}\right)+\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}\right)\left(\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\right)\right]F_{TT}-F=2k^2p_m \quad .(17)$$

The conservation equation can be written as

$$\dot{\rho}_m+\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)(\rho_m+p_m)=0 \qquad (18)$$

For LRS Bianchi type-I we define the average scale factor R, the mean Hubble parameter H and the anisotropy parameter Δ as

$$R=\left(AB^2\right)^{1/3}, \qquad (19)$$

$$H=\frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right), \qquad (20)$$

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2. \qquad (21)$$

It is observed that the isotropic expansion of the universe is obtained for $\Delta=0$, which depends on the values of unknown scale factors and parameters that are involved in the corresponding model [Sharif &Zubair (2010), Sharif & Kausar (2011), Tiwari (2010)].

Using equation (19), equation (15) can be written as

$$T=-9H^2+J \quad , \quad \text{with}$$

$$J=\frac{\dot{A}^2}{A^2}+2\frac{\dot{B}^2}{B^2}, \qquad (22)$$

which can be written as

$$H=\frac{1}{3}\sqrt{J-T}. \qquad (23)$$

Also, for $F(T)=T$, $F_T=1$, equations(16) and (17) will reduce to

$$\rho_m+\rho_T=\frac{1}{2k^2}\left[-4\left(2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right)+T\right] (24)$$

$$p_m+p_T=\frac{1}{2k^2}\left[4\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{\ddot{B}}{B}\right)-T\right], (25)$$

where

$$\rho_T=\frac{1}{2k^2}\left[-4\left(2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right)(1-F_T)+T-F\right], \qquad (26)$$

$$p_T=\frac{1}{2k^2}\left[4\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{\ddot{B}}{B}\right)(1-F_T)\right.$$
$$\left.+16\frac{\dot{B}}{B}\left\{\frac{\dot{B}}{B}\left(\frac{\ddot{A}}{A}-\frac{\dot{A}^2}{A^2}\right)+\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}\right)\left(\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\right)\right\}F_{TT}-T+F\right], \quad (27)$$

are the torsion contribution to the energy density and pressure.

The EoS parameter $\omega=\frac{p}{\rho}$ has different values for relativistic and non-relativistic matters.

From equations (16) and (17), the EoS parameter ω takes the form

$$\omega=-1+\frac{(4\gamma-4\xi)F_T-16\phi F_{TT}}{-4\psi F_T+F}, \qquad (28)$$

where

$$\xi=\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{\ddot{B}}{B}\right), \qquad (29)$$

$$\gamma=\frac{\dot{B}^2}{B^2}+2\frac{\ddot{B}}{B}, \qquad (30)$$

$$\phi=\frac{\dot{B}}{B}\left[\frac{\dot{B}}{B}\left(\frac{\ddot{A}}{A}-\frac{\dot{A}^2}{A^2}\right)+\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}\right)\left(\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\right)\right], \qquad (31)$$

$$\psi=2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}. \qquad (32)$$

4.Construction of some *F(T)* Models using EoS Parameter :

Now we construct *F(T)* models in a different way. Consider equation (28) in the following form

$$16\phi F_{TT}+\left[-4\psi(\omega+1)+4\xi-4\gamma\right]F_T+(\omega+1)F=0. \quad (33)$$

By using different values of EoS parameter ω in equation (33), we consider the following cases and construct different *F(T)* models.

Case-1:

For EoS parameter $\omega = \frac{1}{3}$, equation (33) gives

$$16\phi F_{TT} + \left(4\xi - 4\gamma - \frac{16}{3}\psi\right)F_T + \frac{4}{3}F = 0 \qquad (34)$$

The above equation gives the following general solution

$$F(T) = d_1 \exp T\left[\frac{4\psi - 3\xi + 4\gamma + \sqrt{(3\xi - 4\psi - 3\gamma)^2 - 48\phi}}{24\phi}\right] + d_2 \exp T\left[\frac{4\psi - 3\xi + 4\gamma - \sqrt{(3\xi - 4\psi - 3\gamma)^2 - 48\phi}}{24\phi}\right], \quad (35)$$

where d_1 and d_2 are constants.

Case-2:

In this case we consider $\omega = 0$, equation (33) takes the form

$$16\phi F_{TT} + (-4\psi + 4\xi - 4\gamma)F_T + F = 0. \qquad (36)$$

which gives the general solution in the form

$$F(T) = d_3 \exp T\left[\frac{3\psi - 3\xi + 3\gamma + \sqrt{(-3\psi + 3\xi - 3\gamma)^2 - 36\phi}}{24\phi}\right] + d_4 \exp T\left[\frac{3\psi - 3\xi + 3\gamma - \sqrt{(-3\psi + 3\xi - 3\gamma)^2 - 36\phi}}{24\phi}\right], \qquad (37)$$

where d_3 and d_4 are other constants .

Case-3 :

Now for $\omega = -1$, equation (33) gives

$$16\phi F_{TT} + (4\xi - 4\gamma)F_T = 0. \qquad (38)$$

The above equation has two solutions. $F(T) = d_5$ is the first solution and the second solution becomes

$$F(T) = d_6 \exp\left[\left(\frac{\gamma - \xi}{4\phi}\right)T\right], \quad (39)$$

where d_5 and d_6 are the arbitrary constants.

Conclusion:

In the present paper, we have investigated different types of DE cosmology in *F(T)* gravity. By using EoS parameter we have discussed LRS Bianchi type-I models in*F(T)* gravity.These *F(T)* gravity models represents three different phases such as matter, radiation and DE respectively corresponding to $\omega = 0$, $\omega = 1/3$ and $\omega = -1$. The equations (35), (37) and (39) represents consecutivethree *F(T)* models corresponding to the three phases i.e., radiation, matter and DE respectively. It is found that the universe takes a transition between phantom and non -phantom phases for the exponential form of *F(T)* models[Bamba*et al.* (2011)]. An attempt has been made for the revival of the forms used by Sharif & Rani (2011).Our results are analogous to the results obtained by Sharif and Rani (2011).

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